Exam PPP

11 April 2017

- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Final point grade = total number of points/11 + 1

1. **Pion-nucleon scattering (20 points)**
Consider the scattering of a pion-beam on a proton target, $\pi^+ + p \to \pi^+ + p$. The pion-beam momentum is chosen such that the reaction takes predominantly place via the production and decay of an intermediate Δ resonance, e.g. $\pi^+ + p \to \Delta^{++} \to \ldots$. The Δ resonance has an isospin (I), spin (J) and parity (P) of $I(J^P)$=3/2(3/2$^+$) with a mass of M_Δ=1232 MeV. The isospin/spin/parity of the proton is $I(J^P)$=1/2(1/2)$^+$. The pion has a isospin/spin/parity of $I(J^P)$=1(0$^-$). with a mass of M_p=940 MeV (M_π=140 MeV), respectively.

   a) Historically, the experimental observation of the Δ resonance has led to the theory of Quantum Chromodynamics in which quarks carry a color degree of freedom. Explain how an observation of the Δ gives rise to the need of color assuming (iso)spin symmetry.

   b) Sketch the Feynman diagram at the quark level of the scattering process with an intermediate Δ. Note that the process takes place via the strong interaction.

   c) Calculate the pion momentum that would produce a Δ at its resonance mass.

   d) Use isospin conservation to estimate the cross section ratio

   $$\frac{\sigma(\pi^+ + p \to \pi^+ + p)}{\sigma(\pi^+ + n \to \pi^+ + n)},$$

   assuming that both reactions take place at the same center-of-mass energy and via an intermediate Δ resonance. Motivate your answer.

2. **Charmonium and tetraquarks (20 points)**
The mysterious $X(3872)$ is a particle with a hidden charmonium content (e.g. it contains $c\bar{c}$ pair). It has a mass of 3872 MeV and a spin-(charge) parity of J^{PC}=1^{++}. It was discovered by Belle in 2003, and, more recently, observed by BESIII in the reaction $e^+ + e^- \to \psi(4260) \to \gamma + X(3872)$, where $\psi(4260)$ is a vector charmonium state (J^{PC}=1^{--}) with a mass of 4260 MeV and γ a radiative photon (J^{PC}=1^{--}).

   a) It is extremely rare to produce the $X(3872)$ in the electromagnetic annihilation process $e^+e^- \to X(3872)$. Explain why such a direct production is practically impossible. Motivate your answer.

   b) Most of the $\psi(4260)$ states will decay into a pair of open-charm mesons, $D\bar{D}$. What is the reason on the quark level why such a decay is favorable? Motivate your answer. The mass of a D meson is 1870 MeV and it is composed of a (anti) charm quark together with a (anti) light u or d quark.

   c) The $X(3872)$ cannot decay into two open-charm mesons, $D\bar{D}$ ($J^P(D)$=0^-), which makes its width very small. Argue why this decay is forbidden in the strong interaction.

   d) One of the strongest decay modes of the $X(3872)$ turns out to be $X(3872) \to J/\psi + \rho$, where J/ψ is the lightest vector charmonium state ($c\bar{c}$) and ρ is a meson composed of light (u, d) quarks with an isospin of one. Explain why such an observation favors the interpretation of the $X(3872)$ as a tetraquark state in stead of a conventional $c\bar{c}$ charmonium state. The tetraquark in this case has a quark contents of $q\bar{q}c\bar{c}$ whereby q represents one of the light quarks u or d.

3. **The pion and sigma (20 points)**
The lightest hadron in nature is the pion, π, with J^P=0^- and a mass of 140 MeV. This meson is also known as the (pseudo) Goldstone particle in hadron physics. The chiral-partner state of the pion is the sigma, σ, with J^P=0^+ and a significantly larger mass of about 600 MeV. It is believed that the mass of hadrons originate from a spontaneous chiral-symmetry breaking. On the quark level, the π and σ mesons are composed of $q\bar{q}$ where q=u or d.

   a) What is the origin of chiral symmetry and explain qualitatively the phenomena of spontaneous chiral-symmetry breaking in the context of the pion and sigma fields.

   b) The q and $\bar{q}$ are both spin-1/2 fermions. Analyze the possible sum of the spins and orbital angular momenta between the two quarks in the case of a π and in the case of a σ meson. Motivate your answer.

   c) Electrically charged pions decay via the weak interaction into a (anti) neutrino and a charged lepton. Sketch the Feynman diagram of such a decay.

   d) Explain why the weak decay rate for the pion into a muon+neutrino is much larger than that the decay rate into an electron/positron+neutrino final state. The mass of a pion is 140 MeV, the mass of an electron/positron is 0.5 MeV, and the mass of a muon is 105 MeV. The neutrino mass can be neglected.

4. **Solar neutrinos (20 points)**
Nuclear (fusion) processes that take place in the Sun provide a source of electron neutrinos, ν_e, with a large variation in energies that can go up to at most 20 MeV. The detection of electron neutrinos can take place via Cherenkov radiation of charged leptons in purified water such as the Super-Kamiokande setup in Japan. The (dis)appearance probability of $\nu_e \to \nu_\mu$ can be approximated (assuming oscillations among two neutrino flavors) by $P_{e\mu} = \sin^2(2\theta_{12})\sin^2(1.27\Delta m^2\, L/E)$ with θ_{12} the corresponding mixing angle, Δm^2 the mass-square difference between the two neutrino mass eigenstates in units of eV^2, L the distance from the neutrino source in units of kilometers, and E the neutrino energy in units of GeV. The diameter of the Sun is about $1.4{\times}10^6$ kilometers. The mass-square difference, Δm^2 is about $8{\times}10^{-5}$ eV^2.

a) Sketch the dominant weak processes (neutral and charged currents) that can take place between the electron-neutrino and nucleons or atomic electrons that produce fast-moving electrons in the final state. Why is the detection sensitivity for electron-neutrinos larger than for neutrinos with another flavor?

b) An electron-neutrino survival probability, P_{ee}, of 0.58 has been measured for low energy electron neutrinos; See Fig. 2. Assume that the MSW effect does not play a role in this case (E_ν<2 MeV). Estimate the angle θ_{12} from such an observation. Motivate your answer.

c) The MSW effect will play a role in the neutrino oscillation process for neutrino energies larger than about 2 MeV. The survival probability will significantly be reduced compared to the vacuum conditions as can be observed from Fig. 2. Describe qualitatively the MSW effect and its origin.

d) Neutrino oscillations are only sensitive to the mass-square difference between the neutrino eigenstates. Describe briefly an experimental method that will be able to set an absolute mass scale on the neutrino mass.

5. **Kaons (20 points)**
To study the $K^0-\bar{K}^0$ oscillations, a pure beam of K^0 ($\bar{s}d$) mesons is prepared using the reaction $\pi^-p \to K^0\Lambda$ slightly above its production threshold. The Λ hyperon is the lightest baryon with a strange quark (uds). Kaons have a spin-parity of J^P=0^-.

a) Why are there no $\bar{K}^0$ mesons produced during these π^-p collisions.

b) Sketch the underlying (quark-level) process that can cause $K^0-\bar{K}^0$ oscillations.

c) The K_S and K_L are the mass eigenstates with lifetimes of 90 ps and 50 ns, respectively. In the case of CP invariance, the K_S and K_L are also CP eigenstates. Construct the CP-even (K_S) and CP-odd (K_L) wavefunctions in terms of the K^0 and $\bar{K}^0$ components.

d) The K_S decays predominantly (close to 100%) into two pions ($J^P(\pi)$=0^-). The $K_L \to \pi\pi$ process has a tiny branching fraction of $2{\times}10^{-3}$. Explain this observation by evaluating the CP eigenvalue of the two-pion system.

e) Describe a method to count the intensity of K^0 and $\bar{K}^0$ mesons as function of distance from the production target. Estimate the fraction of K^0 and $\bar{K}^0$ after a few meters from target. Motivate your answers.

43. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2			
m_1 m_2	Coefficients		
⋮ ⋮			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

1/2×1/2

m_1 m_2	1, +1
+1/2 +1/2	1

m_1 m_2	1, 0	0, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	1, −1
−1/2 −1/2	1

1×1/2

m_1 m_2	3/2, +3/2
+1 +1/2	1

m_1 m_2	3/2, +1/2	1/2, +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	3/2, −1/2	1/2, −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	3/2, −3/2
−1 −1/2	1

2×1/2

m_1 m_2	5/2, +5/2
+2 +1/2	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	5/2, +1/2	3/2, +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	5/2, −1/2	3/2, −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	5/2, −3/2	3/2, −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	5/2, −5/2
−2 −1/2	1

2×1

m_1 m_2	3, +3
+2 +1	1

m_1 m_2	3, +2	2, +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	3, +1	2, +1	1, +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	3, 0	2, 0	1, 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	3, −1	2, −1	1, −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	3, −2	2, −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	3, −3
−2 −1	1

1×1

m_1 m_2	2, +2
+1 +1	1

m_1 m_2	2, +1	1, +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	2, 0	1, 0	0, 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	2, −1	1, −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	2, −2
−1 −1	1

3/2×1/2

m_1 m_2	2, +2
+3/2 +1/2	1

m_1 m_2	2, +1	1, +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	2, 0	1, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	2, −1	1, −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	2, −2
−3/2 −1/2	1

3/2×1

m_1 m_2	5/2, +5/2
+3/2 +1	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	5/2, −3/2	3/2, −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	5/2, −5/2
−3/2 −1	1

3/2×3/2

m_1 m_2	3, +3
+3/2 +3/2	1

m_1 m_2	3, +2	2, +2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	−1/2

m_1 m_2	3, +1	2, +1	1, +1
+3/2 −1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	−2/5
−1/2 +3/2	1/5	−1/2	3/10

m_1 m_2	3, 0	2, 0	1, 0	0, 0
+3/2 −3/2	1/20	1/4	9/20	1/4
+1/2 −1/2	9/20	1/4	−1/20	−1/4
−1/2 +1/2	9/20	−1/4	−1/20	1/4
−3/2 +3/2	1/20	−1/4	9/20	−1/4

m_1 m_2	3, −1	2, −1	1, −1
+1/2 −3/2	1/5	1/2	3/10
−1/2 −1/2	3/5	0	−2/5
−3/2 +1/2	1/5	−1/2	3/10

m_1 m_2	3, −2	2, −2
−1/2 −3/2	1/2	1/2
−3/2 −1/2	1/2	−1/2

m_1 m_2	3, −3
−3/2 −3/2	1

2×3/2

m_1 m_2	7/2, +7/2
+2 +3/2	1

m_1 m_2	7/2, +5/2	5/2, +5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	−3/7

m_1 m_2	7/2, +3/2	5/2, +3/2	3/2, +3/2
+2 −1/2	1/7	16/35	2/5
+1 +1/2	4/7	1/35	−2/5
0 +3/2	2/7	−18/35	1/5

m_1 m_2	7/2, +1/2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+2 −3/2	1/35	6/35	2/5	2/5
+1 −1/2	12/35	5/14	0	−3/10
0 +1/2	18/35	−3/35	−1/5	1/5
−1 +3/2	4/35	−27/70	2/5	−1/10

m_1 m_2	7/2, −1/2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1 −3/2	4/35	27/70	2/5	1/10
0 −1/2	18/35	3/35	−1/5	−1/5
−1 +1/2	12/35	−5/14	0	3/10
−2 +3/2	1/35	−6/35	2/5	−2/5

m_1 m_2	7/2, −3/2	5/2, −3/2	3/2, −3/2
0 −3/2	2/7	18/35	1/5
−1 −1/2	4/7	−1/35	−2/5
−2 +1/2	1/7	−16/35	2/5

m_1 m_2	7/2, −5/2	5/2, −5/2
−1 −3/2	4/7	3/7
−2 −1/2	3/7	−4/7

m_1 m_2	7/2, −7/2
−2 −3/2	1

2×2

m_1 m_2	4, +4
+2 +2	1

m_1 m_2	4, +3	3, +3
+2 +1	1/2	1/2
+1 +2	1/2	−1/2

m_1 m_2	4, +2	3, +2	2, +2
+2 0	3/14	1/2	2/7
+1 +1	4/7	0	−3/7
0 +2	3/14	−1/2	2/7

m_1 m_2	4, +1	3, +1	2, +1	1, +1
+2 −1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	−1/14	−3/10
0 +1	3/7	−1/5	−1/14	3/10
−1 +2	1/14	−3/10	3/7	−1/5

m_1 m_2	4, 0	3, 0	2, 0	1, 0	0, 0
+2 −2	1/70	1/10	2/7	2/5	1/5
+1 −1	8/35	2/5	1/14	−1/10	−1/5
0 0	18/35	0	−2/7	0	1/5
−1 +1	8/35	−2/5	1/14	1/10	−1/5
−2 +2	1/70	−1/10	2/7	−2/5	1/5

m_1 m_2	4, −1	3, −1	2, −1	1, −1
+1 −2	1/14	3/10	3/7	1/5
0 −1	3/7	1/5	−1/14	−3/10
−1 0	3/7	−1/5	−1/14	3/10
−2 +1	1/14	−3/10	3/7	−1/5

m_1 m_2	4, −2	3, −2	2, −2
0 −2	3/14	1/2	2/7
−1 −1	4/7	0	−3/7
−2 0	3/14	−1/2	2/7

m_1 m_2	4, −3	3, −3
−1 −2	1/2	1/2
−2 −1	1/2	−1/2

m_1 m_2	4, −4
−2 −2	1

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 1: Clebsch-Gordan coefficients.

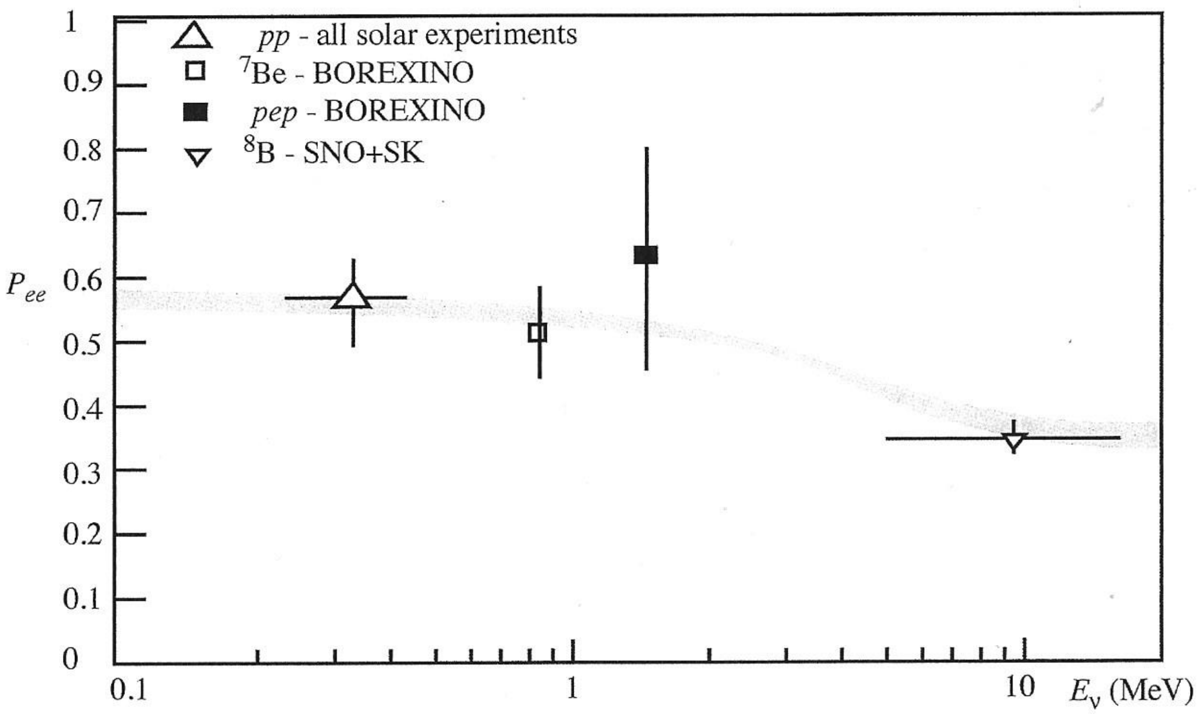

Figure 2: Electron neutrino survival probability at different components of the solar energy spectrum.